$$*/\ \int_2^e \frac{\ln^2 x}{x}dx = \int_2^e \frac{1}{x}.(\ln x)^2 dx$$

$$= \int_2^e (\ln x)'.\ln^2 x dx = \left[\frac{1}{3}\ln^3 x\right]_2^e$$

$$= \frac{1}{3}\ln^3 e - \frac{1}{3}\ln^3 2$$

$$= \frac{1}{3} - \frac{1}{3}\ln^3 2$$

$$*/\ \int_0^{\frac{\pi}{2}} \sin 2x dx = \left[\frac{-\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= -\frac{\cos\pi}{2} + \frac{\cos 0}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

Exercice 2

$$f(x) = \frac{3x-5}{2x-1}$$

$$g(x) = \frac{x^2+2x-5}{x+4}$$

$$h(x) = \frac{e^x-2}{e^x+1}$$

1) $a + \frac{b}{2x-1} = \frac{a(2x-1)+b}{2x-1}$

$$= \frac{2ax+b-a}{2x-1} = \frac{3x-5}{2x-1}$$

D'où : $\begin{cases} 2a=3 \\ b-a=-5 \end{cases} \Leftrightarrow \begin{cases} a=\frac{3}{2} \\ b=a-5 \end{cases} \Leftrightarrow \begin{cases} a=\frac{3}{2} \\ b=-\frac{7}{2} \end{cases}$

ET par suite:

$$f(x) = \frac{3}{2} + \frac{-\frac{7}{2}}{2x-1}$$

D'où:

$$\int_1^2 f(x)dx = \int_1^2 \left(\frac{3}{2} - \frac{7}{2}.\frac{1}{2x-1}\right)dx$$

$$= \int_1^2 \frac{3}{2}dx - \frac{7}{4}\int_1^2 \frac{2}{2x-1}dx$$

$$= \left[\frac{3}{2}x\right]_1^2 - \frac{7}{4}\left[\ln(2x-1)\right]_1^2$$

$$= \left(3-\frac{3}{2}\right) - \frac{7}{4}(\ln 3 - \ln 1) = \frac{3}{2} - \frac{7}{4}\ln 3$$

2) $g(x) = ax+b+\frac{c}{x+4}$

$$= \frac{ax(x+4)+b(x+4)+c}{x+4}$$

$$= \frac{ax^2+4ax+bx+4b+c}{x+4}$$

$$= \frac{ax^2+(4a+b)x+4b+c}{x+4}$$

D'où : $\begin{cases} a=1 \\ 4a+b=2 \\ 4b+c=-5 \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b=2-4a \\ c=5-4b \end{cases}$

$$\Leftrightarrow \begin{cases} a=1 \\ b=2-4=-2 \Rightarrow b=-2 \\ c=-5-4(-2)=-5+8=3 \end{cases}$$

Donc : $\boxed{a=1, b=-2, c=3}$

donc :

$$\int_{-3}^0 g(x)dx = \int_{-3}^0 \left(x-2+\frac{3}{x+4}\right)dx$$

$$= \left[\frac{x^2}{2} - 2x + 3\ln(x+4)\right]_{-3}^0$$

$$= (3\ln 4) - \left(\frac{9}{2} + 6 + 3\ln 1\right)$$

$$= 3\ln 4 - \frac{21}{2}$$

3) $h(x) = a + \frac{be^x}{e^x+1} = \frac{a(e^x+1)+be^x}{e^x+1}$

$$= \frac{ae^x+a+be^x}{e^x+1} = \frac{(a+b)e^x+a}{e^x+1}$$